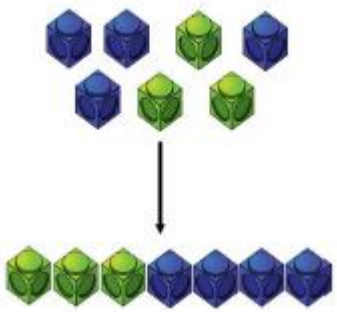
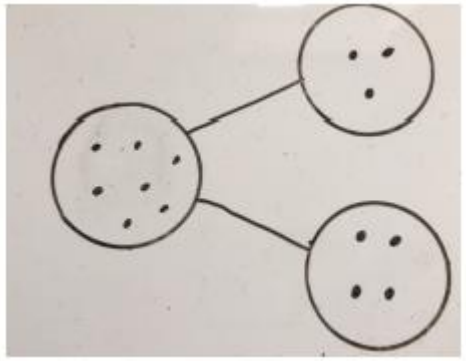
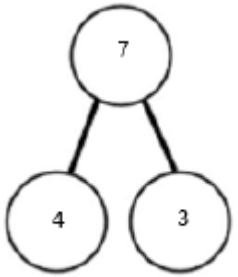
Berry Hill Primary and Nursery School

Progression in Calculations

Addition			
Key Language: sum, total parts and wholes, plus, add, altogether, more, ‘is equal to’ ‘is the same as’.			
Objective and Strategies	Concrete	Pictorial	Abstract
Combining two parts to make a whole: part-whole model	Combining two parts to make a whole (use other resources too. e.g. eggs, shells, teddy bears, cars..) 	Children to represent the cubes using dots or crosses. They could put each part on a part whole model too. 	4 + 3 = 7 Four is a part, 3 is a part and the whole is seven. 

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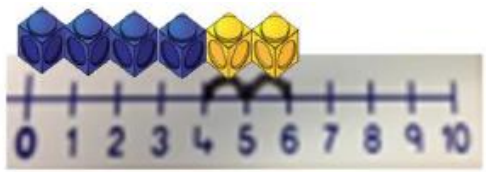
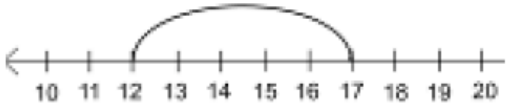
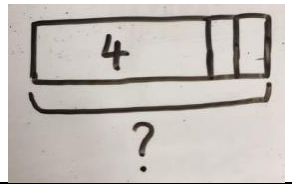
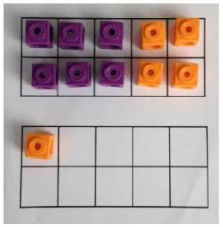
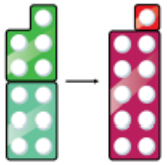
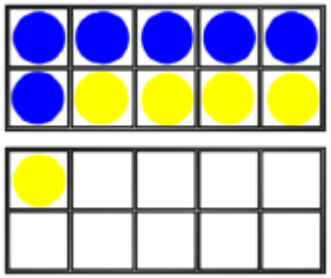
Progression in Calculations

Counting on using number lines	Counting on using number lines using for example cubes or other counters above the line.  	12 + 5 = 17  Start at the larger number on the number line and count on in ones or in one jump to find the answer.  Use of a bar model which encourages the children to count on, rather than to count all.	Children use mental strategies to count on and then record the number sentence. What is 2 more than 4? What is the sum of 2 and 4? What is the total of 4 and 2? 4 + 2
Regrouping to make 10.	Regrouping to make 10 using ten frames and for example counters/cubes or using Numicon. 6 + 5 = 11 Start with the bigger number and use the smaller number to make 10.  	Children to draw the ten frame and counters/cubes. 	Children visualise regrouping mentally and form the number sentence. 6 + 5 = 10 + 1 = 11 = 5 + 1 + 5 = 11 7 + 6 = 10 + 3 = 13

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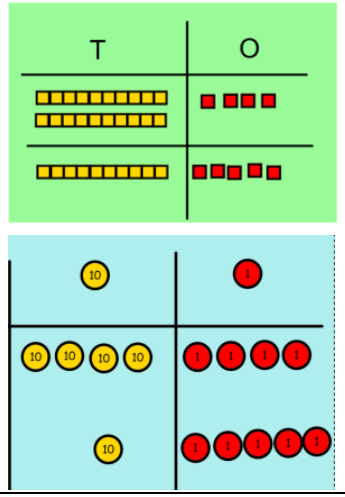
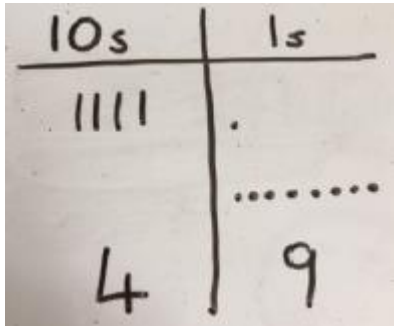
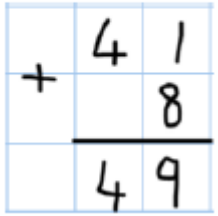
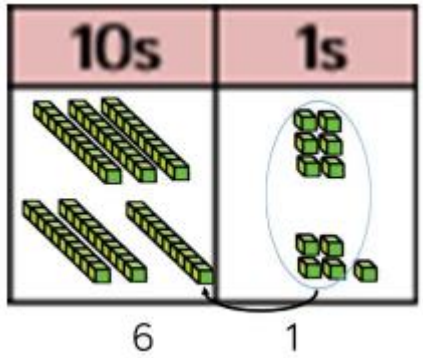
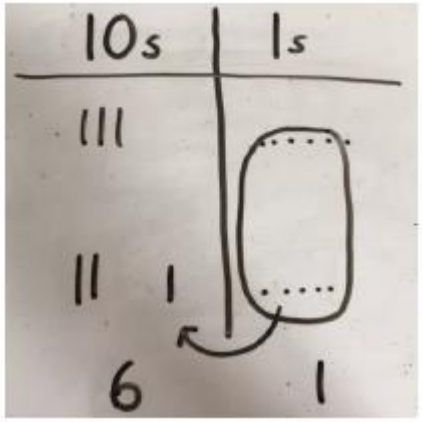
Progression in Calculations

TO+O - Column method- no regrouping	24 + 15= Add together the ones first then add the tens. Use the Base 10 blocks first before moving onto place value counters.  	After practically using the base 10 blocks and place value counters, children can draw the counters to help them to solve additions. Children to represent the base 10 e.g. lines for tens and dot/crosses for ones. This could also be represented in a part, part, part, whole picture i.e. 40 + 1 + 8 = , or by drawing place value counters. 41 + 8 =  	41 + 8 Children picture use of partitioning and place value in their mind's eye and record in number sentences or in a formal written layout. (no bridging of 10). 1 + 8 = 9 40 + 9 = 49 
TO + TO Column method - regrouping	TO + TO using base 10. Continue to develop understanding of partitioning and place value. 36 + 25  	Children to represent the base 10 in a place value chart  	Looking for ways to make 10. 36 + 25= 30 + 20 = 50 5 + 5 = 10 50 + 10 + 1 = 61 1 5 36 Formal method: +25 61 1 This can be displayed either with a space for carries under the numbers being added (abacus) or with carries below depending on the needs of the pupils. If they continually forget their carries use abacus layout.

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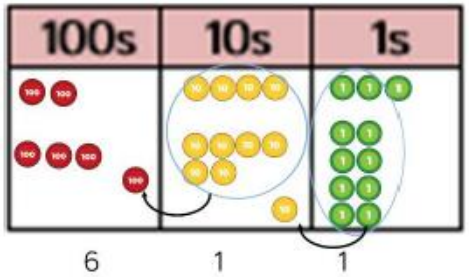
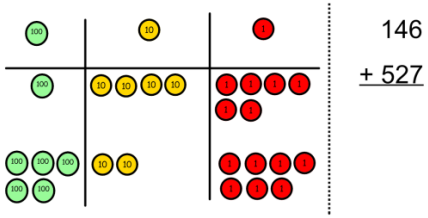
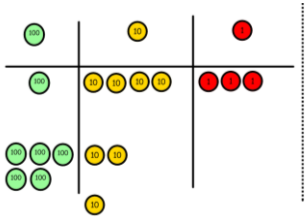
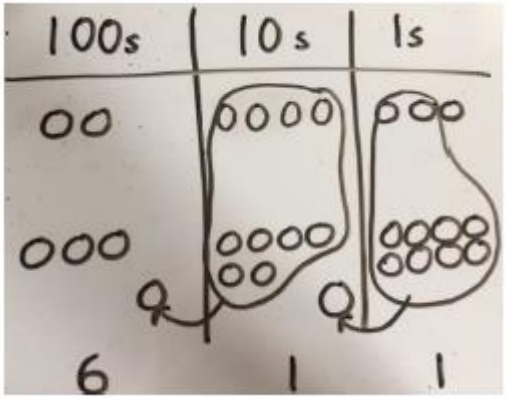
Progression in Calculations

HTO + TO, HTO + HTO etc. Column method- regrouping	Use of place value counters to add HTO + TO, HTO + HTO etc. When there are 10 ones in the 1s column we exchange them for 1 ten, when there are 10 tens in the 10s column we exchange for 1 hundred.  Make both numbers on a place value grid.  146 + 527 Add up the units and exchange 10 ones for one 10.  146 + 527 Add up the rest of the columns, exchanging the 10 counters from one column for the next place value column until every column has been added. This can also be done with Base 10 to help children clearly see that 10 ones equal 1 ten and 10 tens equal 100. PV counters may also be used.	Children can draw a pictoral representation of the columns and place value counters to further support their learning and understanding, circling when they make an exchange.  	Children use a formal written method. 243 +368 611 1 1 Carries can be placed in a separate line above the answer line as demonstrated in abacus examples if required. As the children move on, introduce decimals with the same number of decimal places and different. Money can be used here. 72.8 + 54.6 127.4 1 1 £ 2 3 . 5 9 + £ 7 . 5 5 £ 3 1 . 1 4 1 1 1

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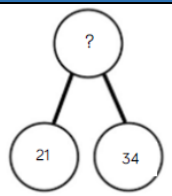
Progression in Calculations

Examples of conceptual variation; different ways to ask children to solve 21 + 34



?	
21	34

Word problems:
In year 3, there are 21 children and in year 4, there are 34 children.
How many children in total?

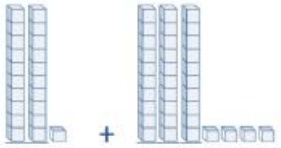
21 + 34 = 55. Prove it

21
+34
___

21 + 34 =

☐ = 21 + 34

Calculate the sum of twenty-one and thirty-four.



Missing digit problems:

10s	1s
10 10	1
10 10 10	?
?	5

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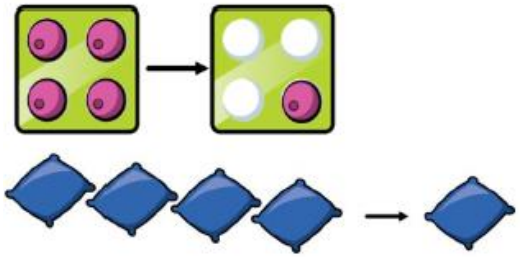
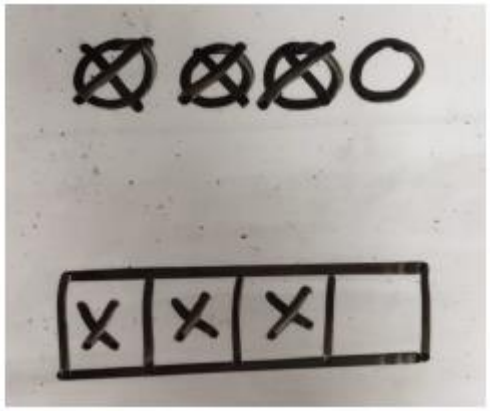
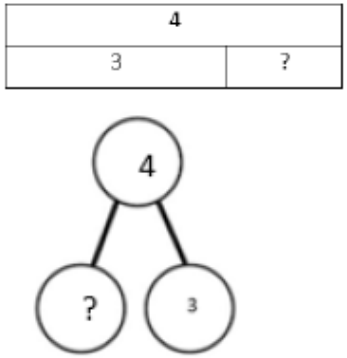
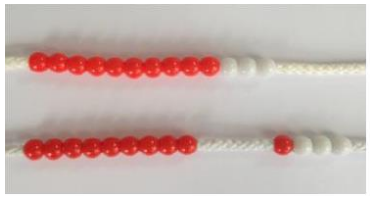
Progression in Calculations

Subtraction			
Key Language: take away, less than, the difference, subtract, minus, fewer, decrease.			
Objective and Strategies	Concrete	Pictorial	Abstract
Taking away ones	Physically taking away and removing objects from a whole (ten frames, Numicon, cubes and other items such as beanbags) 4 – 3 = 1 	Children to draw the concrete resources they are using and cross out the correct amount. The bar model can also be used. 	4- 3 = ☐ = 4 – 3 
Counting back	Make the larger number in your subtraction. Move the beads along your bead string as you count backwards in ones. 13 – 4 	Children to represent what they see pictorially e.g.	Children count back mentally and form the number sentence.

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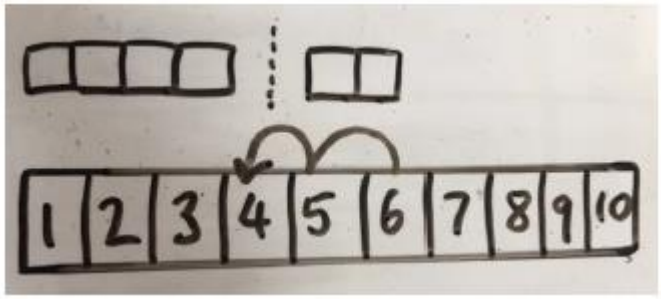
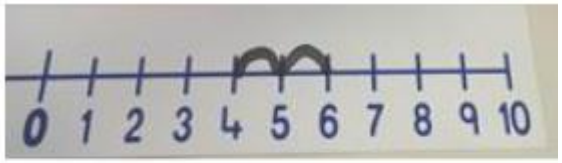
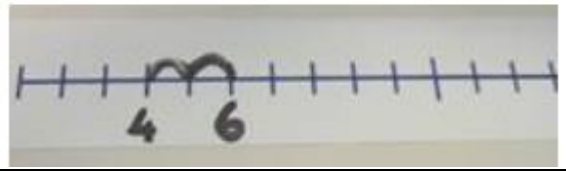
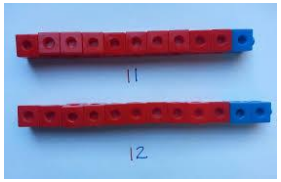
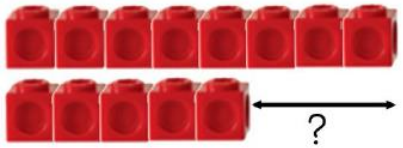
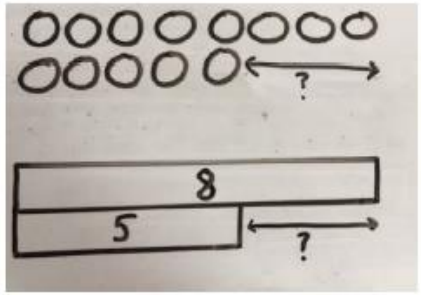
Progression in Calculations

	Use counters and move them away from the group as you take them away counting backwards as you go. 	 Children to represent the calculation on a number line or number track and show their jumps. Encourage children to use an empty number line.   	
Finding the difference	Compare amounts and objects to find the difference.  Use cubes to build towers or make bars to find the difference  	Children to draw the cubes/other concrete objects which they have used or use the bar model to illustrate what they need to calculate.  	Find the difference between 8 and 5. 8 – 5 = The difference is ___ Children to explore why 9 - 6 = 8 – 5 = 7 – 4 have the same difference.

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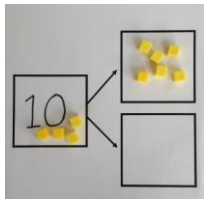
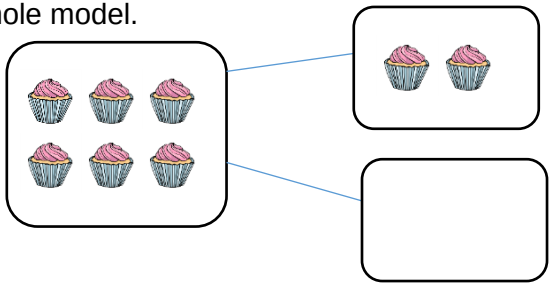
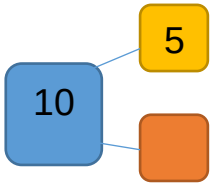
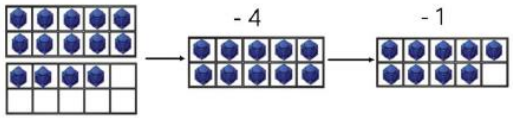
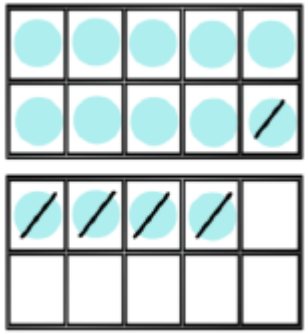
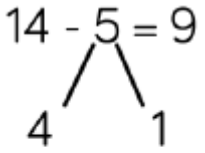
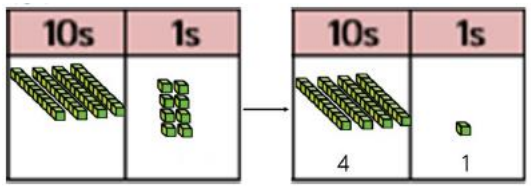
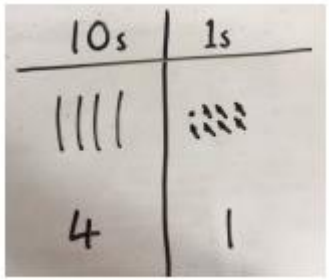
Progression in Calculations

Part Part Whole Model	 Link to addition- use the part whole model to help explain the inverse between addition and subtraction. If 10 is the whole and 6 is one of the parts. What is the other part? 10 - 6 =	Use a pictorial representation of objects to show the part, part, whole model. 	 Move to using numbers within the part whole model.
Bridging 10 using known number facts	Bridge 10 using known number facts with cubes on 10 frames. 14 – 5 	Children to present the ten frame pictorially and discuss what they did to bridge 10.  14 – 5 = 14 – 4 – 1 =	Children to show how they can make 10 by partitioning the subtrahend.  14 – 4 = 10 10 – 1 = 9
Column method without regrouping / exchanging	Column method using concrete resources e.g. base 10 48 – 7 	Children to represent the concrete example pictorially. E.g. 	Fluent use of column method. 

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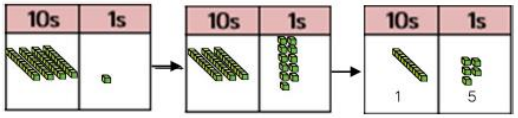
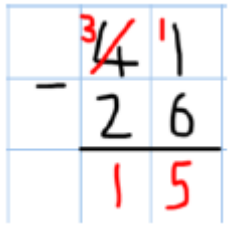
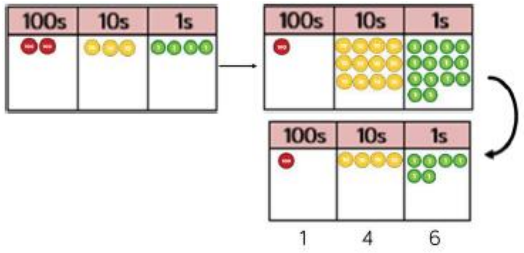
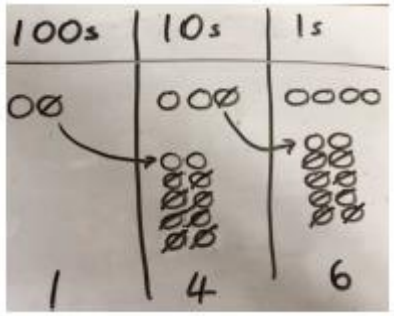
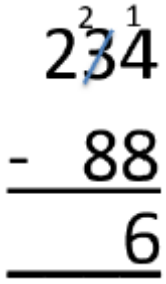
Progression in Calculations

Column method with regrouping / exchanging	Children represent the column method using concrete resources such as base 10 and having to exchange. 41 – 26 	Children represent the concrete pictorially, remembering to show the exchange. 	Formal column method is used. Children must understand that when they have exchanged the 10 they still have 41 because 41 = 30 + 11  Moving on, children have an understanding of subtracting any number including decimals.
Column method using place value counters.	When dealing with bigger numbers, children can use place value counters for representation. 234 – 88 	They move on to representing the place value counters pictorially remembering to show what has been exchanged. 	Children must understand what has happened when they have crossed out digits. 

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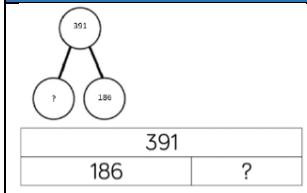
Progression in Calculations

Examples of conceptual variation; different ways to ask children to solve 391 – 186



Raj spent £391, Timmy spent £186.
How much more did Raj spend?

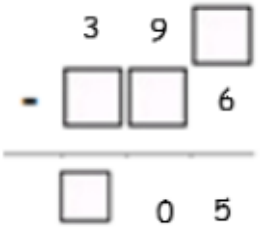
Calculate the difference between 391 and 186.

□ = 391 – 186

391
-186
———

What is 186 less than 391?

Missing digit calculations



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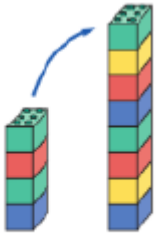
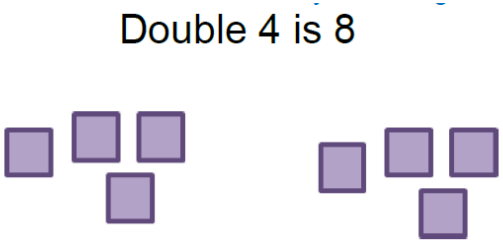
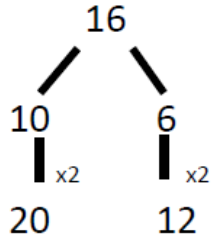
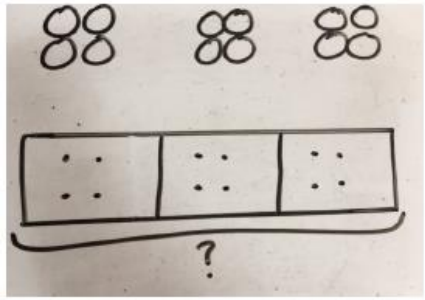
Progression in Calculations

Multiplication

Key Language: double, times, multiplied by, the product of, groups of, lots of, equal groups.

Objective and Strategies	Concrete	Pictorial	Abstract
Doubling	Use practical activities to show how to double a number.  double 4 is 8 4 × 2 = 8	Draw pictures to show how to double a number.  	Partition a number and then double each part before recombining it back together. 
Counting in multiples Repeated grouping / repeated addition	 Count in multiples supported by concrete objects in equal groups. 4 x 2 = 4 lots of 2 2 + 2 + 2 + 2 There are 4 equal groups with 2 in each.	Children represent the practical resources in a picture and use a bar model. 3 x 4 = 3 lots of 4 = 4 + 4 + 4  	Count in multiples of a number aloud. Write sequences with multiples of numbers. 2, 4, 6, 8, 10 5, 10, 15, 20, 25 , 30 Children no longer need concrete or pictorial to complete multiplication. 3 x 4 = 12 4 + 4 + 4 = 12

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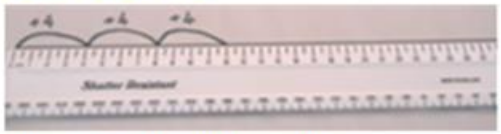
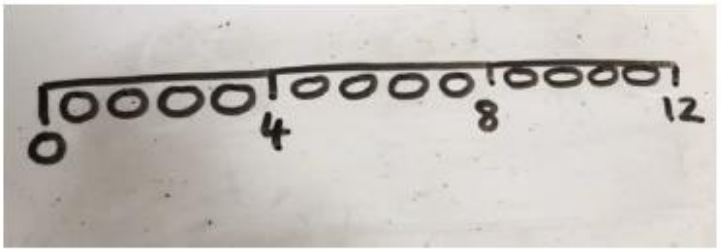
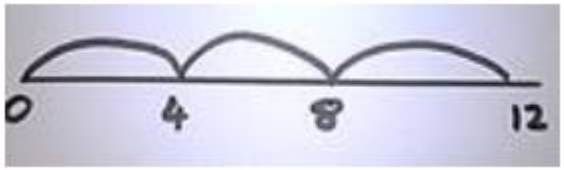
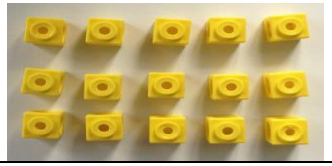
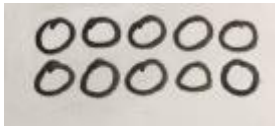
Progression in Calculations

Number lines to show repeated groups	Children place concrete resources above numberlines. 3 x 4   Cuisenaire rods can be used too.	Use of concrete is represented pictorially.  Children move on to using empty number lines. 	Children calculate the multiplication mentally and record the number sentence. 3 x 4 = 12
Arrays- showing commutative multiplication	Create arrays using counters/ cubes to show multiplication sentences.  	Draw arrays in different rotations to find **commutative** multiplication sentences. 5 x 2 = 10  2 x 5 = 10 	Use an array to write multiplication sentences and reinforce repeated addition. 5 + 5 + 5 = 15 3 + 3 + 3 + 3 + 3 = 15 5 x 3 = 15 3 x 5 = 15

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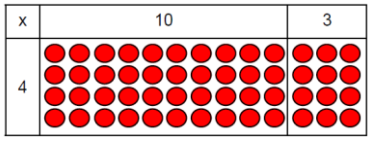
Progression in Calculations

Grid Method

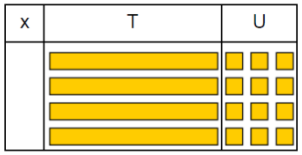
(to be used to develop number sense before moving to formal written methods)

Show the link with arrays to first introduce the grid method.



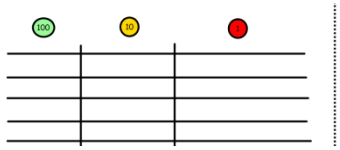
4 rows of 10
4 rows of 3

Move on to using Base 10 to move towards a more compact method.

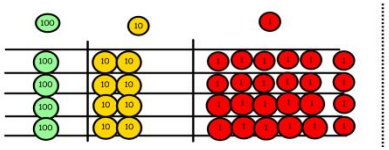


4 rows of 13

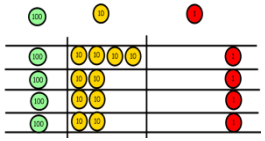
Move on to place value counters to show how we are finding groups of a number.We are multiplying by 4 so we need 4 rows.



Fill each row with 126.



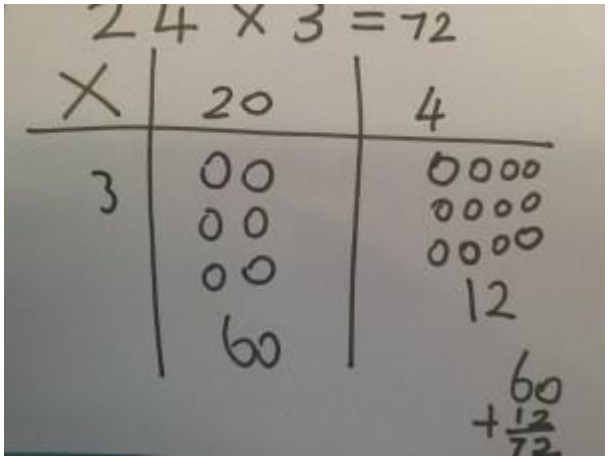
Add up each column, starting with the ones making any exchanges needed.



Then you have your answer.

Children can represent the work they have done with place value counters in a way that they understand.

They can draw the counters, using colours to show different amounts or just use circles in the different columns to show their thinking as shown below.



Start with multiplying by one digit numbers and showing the clear addition alongside the grid.

×	30	5
7	210	35

210 + 35 = 245

Moving forward, multiply by a 2 digit number showing the different rows within the grid method.

	10	8
10	100	80
3	30	24

X	1000	300	40	2
10	10000	3000	400	20
8	8000	2400	320	16

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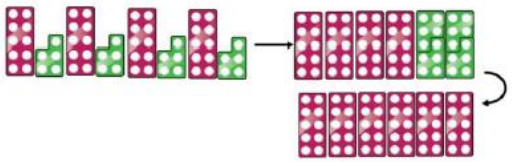
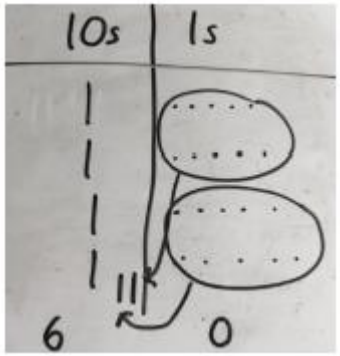
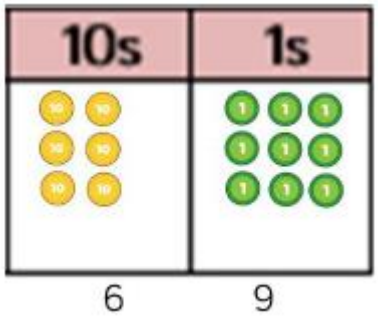
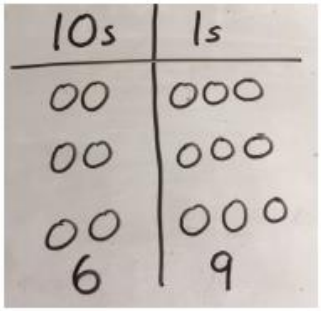
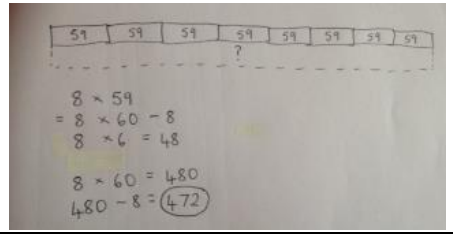
Progression in Calculations

Partition to multiply	Use Numicon, base 10 or Cuisenaire rods. 4 x 15 	Children to represent the concrete manipulatives pictorially. 	Children to be encouraged to show the steps they have taken. 4 x 15 10 5 4 x 10 = 40 4 x 5 = 20 40 + 20 = 60
Formal column method without grouping	Children can continue to be supported by place value counters at the stage of multiplication. 	Children to represent the counters pictorially.  Bar modelling and number lines can support learners when solving problems with multiplication alongside the formal written methods. 	Children to record what it is they are doing to show understanding. 3 × 23 20 3 3 × 20 = 60 3 × 3 = 9 60 + 9 = 69 23 × 3 69

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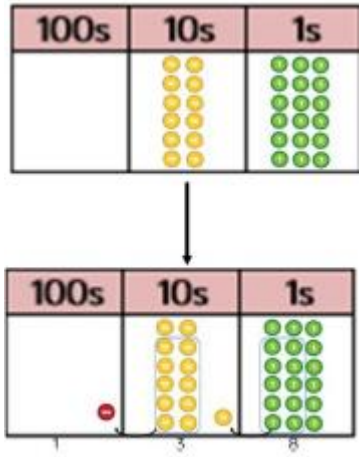
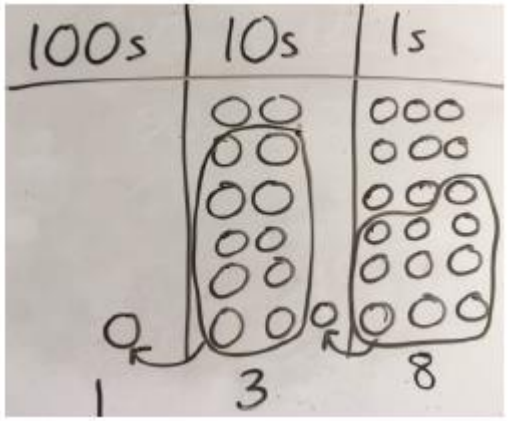
Progression in Calculations

<table>
<tr>
<td>Formal column method with grouping</td>
<td>Formal column method with place value counters.
6 x 23
 
</td>
<td>Children to represent the counters/base 10 pictorially e.g. image below.
 
</td>
<td>Formal written method.
6 x 23 =
23
× 6
138
1 1</td>
</tr>
<tr>
<td colspan="3">When children start to multiply 3d x 3d and 4d x 2d etc., they should be confident with the abstract:
To get 744 children have solved 6 x 124
To get 2480 they have solved 20 x 124

For consistency we teach multiplying by ones first then tens, hundreds etc.</td>
<td>1 2 4
× 2 6
7 4 4
2 4 8 0
3 2 2 4
1 1
Answer: 3224</td>
</tr>
</table>

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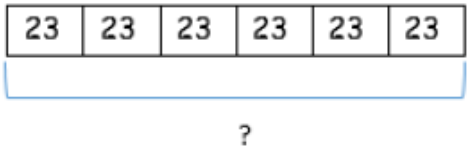
Progression in Calculations

Examples of conceptual variation; different ways to ask children to solve 6 x 23

	Mai had to swim 23 lengths, 6 times a week. How many lengths did she swim in one week? With the counters, prove that 6 x 23 = 138	Find the product of 6 and 23 6 × 23 = □ = 6 × 23 6 × 23 23 × 6	What is the calculation? What is the product? 

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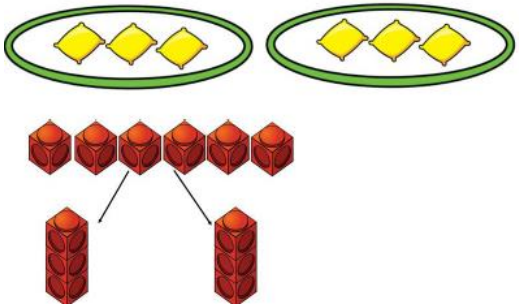
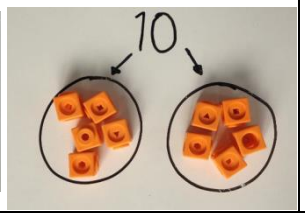
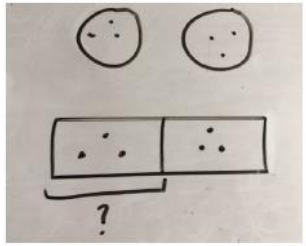
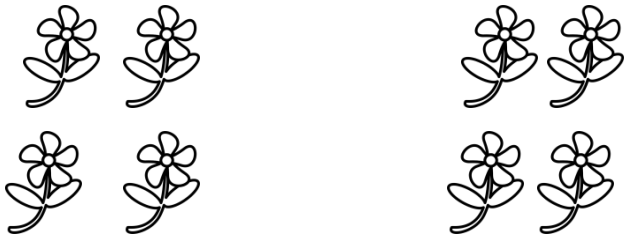
Progression in Calculations

Division			
Key Language: share, group, divide, divided by, half.			
Objective and Strategies	Concrete	Pictorial	Abstract
Sharing objects into groups	Sharing using a range of objects. 6 ÷ 2  I have 10 cubes, can you share them equally in 2 groups?  	Children represent sharing pictorially.   8 ÷ 2 = 4	Share 9 buns between three people. 9 ÷ 3 = 3 Children should also be encouraged to use their times tables facts.

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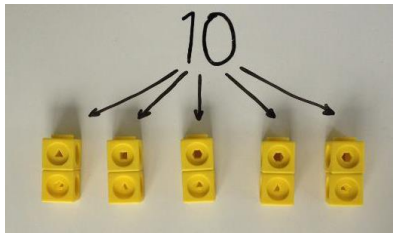
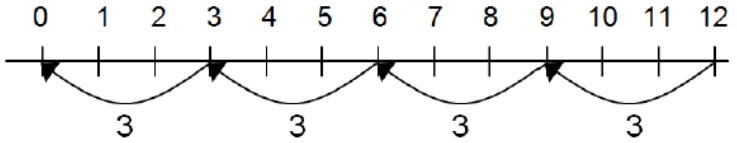
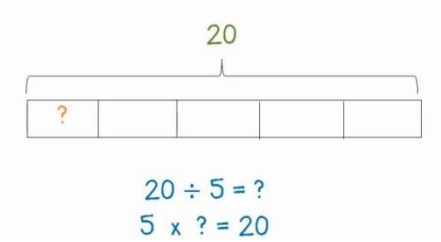
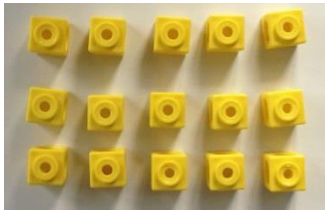
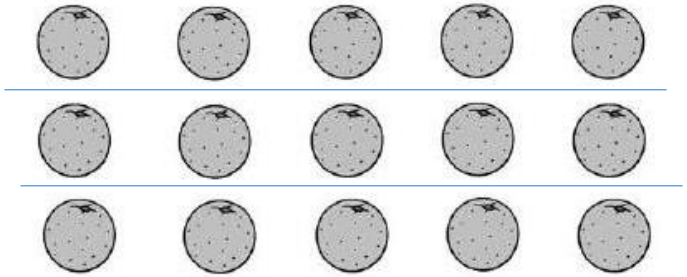
Progression in Calculations

Division as grouping	Divide quantities into equal groups. Use cubes, counters, objects or place value counters to aid understanding. 	Use a number line to show jumps in groups. The number of jumps equals the number of groups.  Think of the bar as a whole. Split it into the number of groups you are dividing by and work out how many would be within each group. 	Children can visualise grouping without concrete or pictorial support and develop knowledge of number fluency to be able to recall facts. 28 ÷ 7 = 4 Divide 28 into 7 groups. How many are in each group?
Division within arrays	 Link division to multiplication by creating an array and thinking about the number sentences that can be created. Eg 15 ÷ 3 = 5 5 x 3 = 15 15 ÷ 5 = 3 3 x 5 = 15	 Draw an array and use lines to split the array into groups to make multiplication and division sentences.	Find the inverse of multiplication and division sentences by creating four linking number sentences. 7 x 4 = 28 4 x 7 = 28 28 ÷ 7 = 4 28 ÷ 4 = 7

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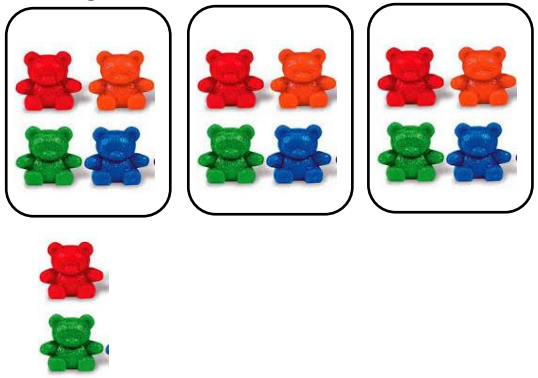
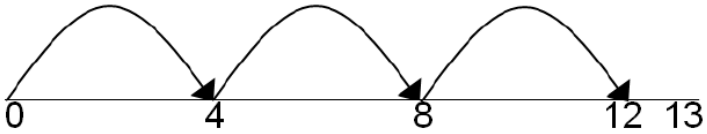
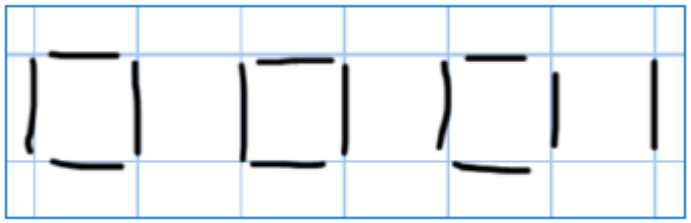
Progression in Calculations

2d ÷ 1d with a remainder	Divide objects between groups and see how much is left over 14 ÷ 3 =  13 ÷ 4 = Use of lollipop sticks to form wholes – squares are made because we are dividing by 4.  There are 3 whole squares, with 1 left over.	Jump forward in equal jumps on a number line then see how many more you need to jump to find a remainder.  Draw dots and group them to divide an amount and clearly show a remainder.  Children to represent the lollipop sticks pictorially.  There are 3 whole squares, with 1 left over	Complete written divisions and show the remainder using r.  13 ÷ 4 = 3 remainder 1 Children should be encouraged to use their times tables facts. ‘3 groups of 4, with 1 left over’.

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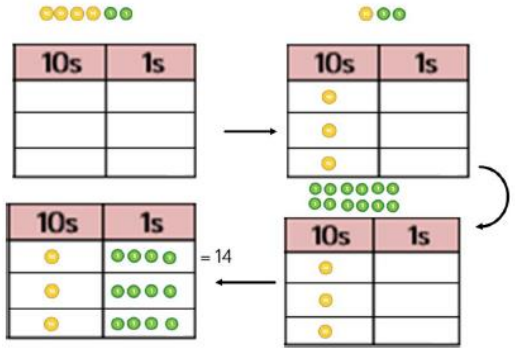
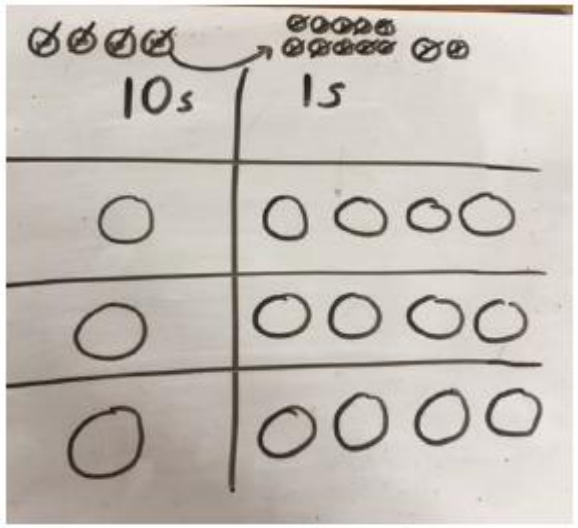
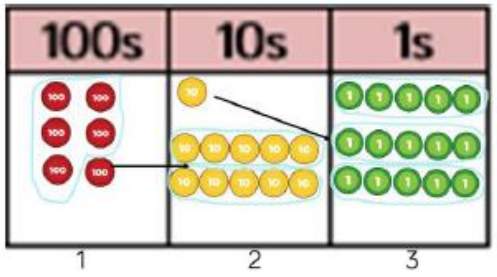
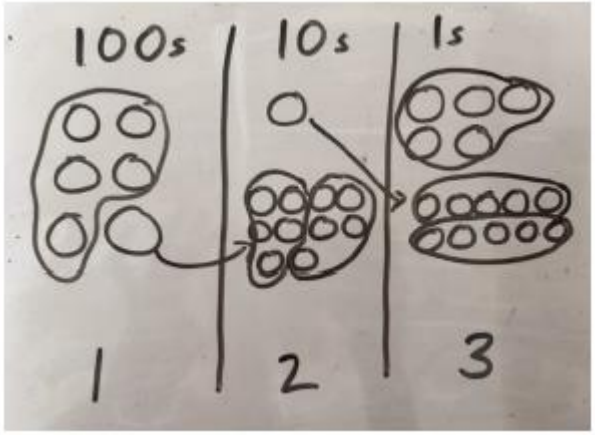
Progression in Calculations

Sharing using place value counters.	42 ÷ 3 = 14 	Children to represent the place value counters pictorially. 	Children to be able to make sense of the place value counters and write calculations to show the process. 42 ÷ 3 42 = 30 + 12 30 ÷ 3 = 10 12 ÷ 3 = 4 10 + 4 = 14
Short division	Children use place value counters to group.  1. Make 615 with place value counters. 2. How many groups of 5 hundreds can you make with 6 hundred counters? 3. Exchange 1 hundred for 10 tens. 4. How many groups of 5 tens can you make with 11 ten counters? 5. Exchange 1 ten for 10 ones. 6. How many groups of 5 ones can you make with 15 ones?	Children represent the place value counters pictorially. 	Children to complete the calculation using the short division scaffold. 123 5 ⟌ 6¹1¹5 Children can prove why this works with counters.

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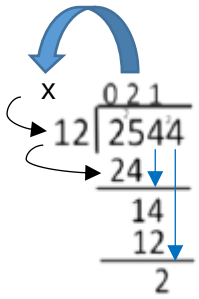
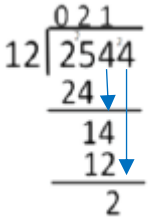
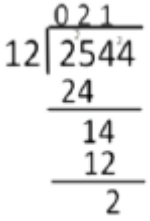
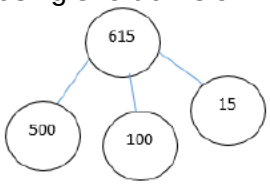
Progression in Calculations

Long division	Once secure with short division children use following steps to long division. (concrete/pictorial do not apply). First use a step by step guide with X and – signs clearly displayed along with arrows showing the numbers being taken down. 	Remove guide but leave arrows for digits that need to be taken down. 	No guide required. Children can move on to working out decimal remainders and remainders as fractions. 

Examples of conceptual variation; different ways to ask children to solve 615 ÷ 5

Using the part whole model below, how can you divide 615 by 5 without using short division? 	I have £615 and share it equally between 5 bank accounts. How much will be in each account? 615 pupils need to be put into 5 groups. How many will be in each group?	$5 \overline{)615}$ 615 ÷ 5 = □ = 615 ÷ 5	What is the calculation? What is the answer? 

Developed in collaboration with Jane Gill – Primary Team maths specialist - from the White Rose Calculation Policy.